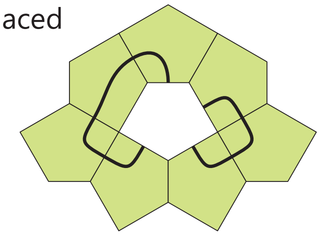
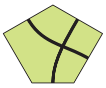
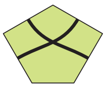
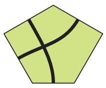
3 points problems

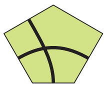
01. A pattern is made of equal pentagons. Which of the tiles below, when placed in the central hole, will form a self-intersecting loop?

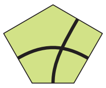


(A) 

(B) 

(C) 

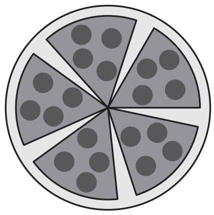
(D) 

(E) 

02. Which of these integers is two less than a multiple of ten, two more than a square, and two times a prime?

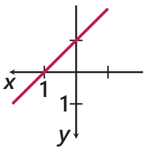
(A) 78 **(B)** 58 **(C)** 38 **(D)** 18 **(E)** 6

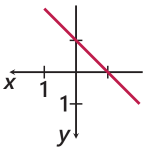
03. A young kangaroo cut a pizza into six equal slices. After eating one slice, he arranged the remaining slices with equal gaps between slices. What size is the angle of each gap?

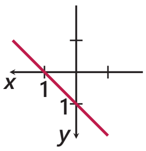


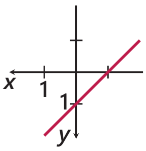
(A) 5° **(B)** 8° **(C)** 9° **(D)** 10° **(E)** 12°

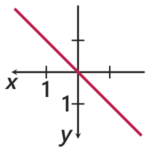
04. Juuso has an unusual habit of drawing the xy-plane with the positive coordinate axes pointing left and down, as shown in the alternatives below. What would the graph of the equation $y = x + 1$ look like in a coordinate system drawn by Juuso?

(A) 

(B) 

(C) 

(D) 

(E) 

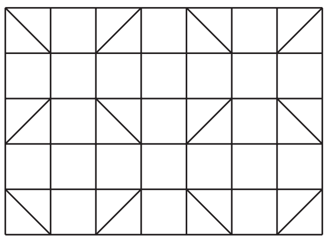
05. Kaito has manipulated a die. The probabilities of rolling a 2, 3, 4 or 5 are still $\frac{1}{6}$ each, but the probability of rolling a 6 is twice the probability of rolling a 1. What is the probability of rolling a 6?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{7}{36}$ **(D)** $\frac{2}{9}$ **(E)** $\frac{5}{18}$

06. Which of the options below has the same value as $16^{15} + 16^{15} + 16^{15} + 16^{15}$?

(A) 16^{19} **(B)** 4^{31} **(C)** 4^{60} **(D)** 16^{60} **(E)** 4^{122}

07. Beaver wishes to color the squares and triangles of the figure on the right so that no two neighbouring figures, even those sharing a single vertex, are the same color. What is the least number of colors needed?



(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

08. There are 6 glasses on a table with their open ends up. In any one move, we turn over exactly 4 of them. What is the least number of moves required to have all glasses upside down?

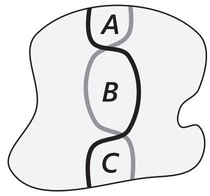
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

09. A student started with the number 1 and multiplied it by either 6 or 10. He then multiplied the result by either 6 or 10 and continued this procedure many times. Which of the following **CANNOT** be one of the numbers he obtained?

(A) $2^{100} \cdot 3^{20} \cdot 5^{80}$ **(C)** $2^{90} \cdot 3^{20} \cdot 5^{70}$ **(E)** $2^{50} \cdot 5^{50}$

(B) $2^{90} \cdot 3^{20} \cdot 5^{80}$ **(D)** $2^{110} \cdot 3^{80} \cdot 5^{30}$

10. A black trail and a grey trail cross a park, as shown. Each trail divides the park into two regions of equal area. Which of the following must be true about the areas A, B and C?



(A) $A = C$ **(C)** $B = \frac{1}{2}(A + C)$ **(E)** $B = \frac{3}{5}(A + C)$

(B) $B = A + C$ **(D)** $B = \frac{2}{3}(A + C)$

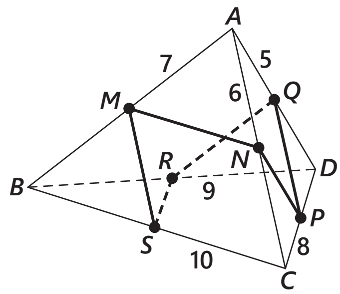
4 points problems

11. Exactly one of these statements about a certain positive integer n is true. Which statement is true?

(A) n is divisible by 3 **(C)** n is odd **(E)** n is prime

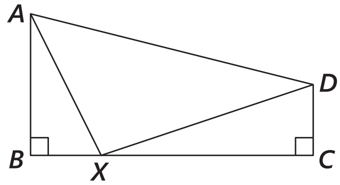
(B) n is divisible by 6 **(D)** $n = 2$

12. A triangular pyramid $ABCD$ has sides of length 5, 6, 7, 8, 9 and 10. The points M, N, P, Q, R and S are the midpoints of the edges of the pyramid, as shown. What is the perimeter of the closed hexagonal line $MNPQRSM$?



(A) 19 **(B)** 20 **(C)** 21 **(D)** 22 **(E)** 23

13. A quadrilateral $ABCD$ has two right angles at B and C, where $AB = 4$, $BC = 8$ and $CD = 2$. Point X lies on BC. What is the minimum value of $AX + DX$?

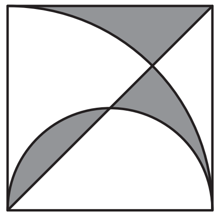


(A) $9\sqrt{2}$ **(B)** 12 **(C)** 13 **(D)** 10 **(E)** 8

14. John has a number of all black or all white unit cubes and wants to build a $3 \times 3 \times 3$ cube using 27 of them. He wants the surface to be exactly half black and half white. What is the smallest number of black cubes he can use?

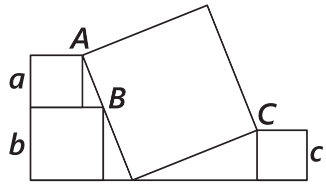
(A) 14 **(B)** 13 **(C)** 12 **(D)** 11 **(E)** 10

15. A diagonal, a semicircle, and a quarter of a circle are drawn in a square of side 6 cm, as shown. What is the area, in cm^2, of the shaded part?



(A) 9 **(B)** 3π **(C)** $6\pi - 9$ **(D)** $\frac{10\pi}{3}$ **(E)** 12

16. The figure shows four squares. The smaller ones have side lengths a, b and c. The vertices A and C of two of the smaller squares coincide with two diagonally opposite vertices of the large square. The vertex B of the third small square is on the side of the large one. Which of the following expressions represents the side length of the largest square?



(A) $\frac{1}{2}(a + b + c)$ **(B)** $\sqrt{a^2 + b^2 + c^2}$ **(C)** $\sqrt{(a + b)^2 + c^2}$ **(D)** $\sqrt{(b - a)^2 + c^2}$ **(E)** $\sqrt{a^2 + ab + b^2 + c^2}$

17. We have two positive numbers p and q, with $p < q$. Which of these expressions is the largest?

(A) $\frac{p + 3q}{4}$ **(B)** $\frac{p + 2q}{3}$ **(C)** $\frac{p + q}{2}$ **(D)** $\frac{2p + q}{3}$ **(E)** $\frac{3p + q}{4}$

18. How many three-digit numbers are there that contain at least one of the digits 1, 2 or 3?

(A) 27 **(B)** 147 **(C)** 441 **(D)** 557 **(E)** 606

19. I write down a 4-digit non-zero number $N = \overline{pqrs}$. When I place a decimal point between the q and the r, I find that the resulting number $\overline{pq,rs}$ is the average of the two-digit numbers $\overline{pq}$ and $\overline{rs}$. What is the sum of the digits of N?

(A) 14 **(B)** 18 **(C)** 21 **(D)** 25 **(E)** 27

20. Two candles of equal length start burning at the same time. One of the candles will burn down in 4 hours, the other in 5 hours, each at their own constant rate. How many hours will they have to burn before one candle is 3 times the length of the other?

(A) $\frac{40}{11}$ **(B)** $\frac{45}{12}$ **(C)** $\frac{63}{20}$ **(D)** 3 **(E)** $\frac{47}{14}$

5 points problems

21. Andre has six cards with one number written on each side of each card. The pairs of numbers on the cards are (5, 12), (3, 11), (0, 16), (7, 8), (4, 14) and (9, 10). The cards can be placed in any order in the blank spaces of the figure.

$$\square + \square + \square - \square - \square - \square = ?$$

What is the smallest result he can get?

(A) −23 **(B)** −24 **(C)** −25 **(D)** −26 **(E)** −27

22. Kangaroo solves the equation $ax^2 + bx + c = 0$, and Beaver solves the equation $bx^2 + ax + c = 0$, where a, b and c are pairwise distinct non-zero integers. It turns out that the equations share a solution. Which of the following must be true?

(A) The common solution must be 0.

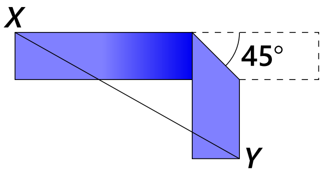
(B) The quadratic equation $ax^2 + bx + c = 0$ has exactly one real solution.

(C) $a > 0$

(D) $b < 0$

(E) $a + b + c = 0$

23. I have a strip of paper that is 12 cm long and 2 cm wide. I make a crease across it at 45° and then fold it, so that the two parts of the strip are aligned in a right angle, as shown. What is the smallest possible length, in cm, of ***XY***?



(A) $6\sqrt{2}$ **(B)** $7\sqrt{2}$ **(C)** 10 **(D)** 8 **(E)** $6 + \sqrt{2}$

24. Rasika has several unbiased 12-sided dice, each with faces labelled 1 to 12. When rolling all the dice at once, the probability of rolling a 12 exactly once is equal to the probability of rolling no 12s. How many dice does Rasika have?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

25. A polynomial $p(x)$ satisfies the relation $p(x + 1) = x^2 - x + 2p(6)$ for every real x. What is the sum of the coefficients of $p(x)$?

(A) -40 **(B)** -6 **(C)** 12 **(D)** 40 **(E)** -20

26. The values of x, y and z satisfy $2^x = 3$, $2^y = 7$ and $6^z = 7$. Which of the following gives the accurate relationship between x, y and z?

(A) $z = \dfrac{y}{1 + x}$

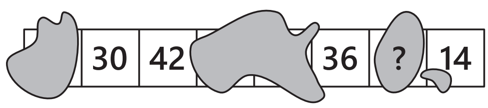
(B) $z = \dfrac{x}{y} + 1$

(C) $z = \dfrac{y}{x} - 1$

(D) $z = \dfrac{x}{y - 1}$

(E) $z = y - \dfrac{1}{x}$

27. A strip of paper consists of eight squares. Initially each square contains the number 0. In every move, we chose four consecutive squares and add 1 to each of the numbers in those squares. The figure on the right shows the outcome after a number of moves but unfortunately some ink is covering some of the squares. What number is written on the square with the question mark?



(A) 24 **(B)** 30 **(C)** 36 **(D)** 42 **(E)** 48

28. A function $f = R \to R$ satisfies $f(20 - x) = f(22 + x)$ for all real x. It is known that f has exactly two roots. What is the sum of these two roots?

(A) –1 **(B)** 20 **(C)** 21 **(D)** 22 **(E)** 42

29. Twelve points are equally spaced on a circumference. How many triangles containing a 45° angle can be formed by choosing three of these points?

(A) 48 **(B)** 60 **(C)** 72 **(D)** 84 **(E)** 96

30. A special four-digit number $\overline{abcd}$ satisfies the equation $\overline{abcd} = a^a + b^b + c^c + d^d$. What is the value of a?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6